

# Forward-Reverse Error Mitigation Procedure for Quantum Annealers

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Qubits North America 2019



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## Funding and Legalese

This material was initiated based on research sponsored by the Air Force Research Laboratory under agreement number FA8750-18-1-0096.



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PH 4990/6990  
ECE 4990/6990

*Introduction to Quantum Computing*

Fall Semester 2019

Mark A. Novotny  
Yaroslav Koshka

D:WAVE 

Take the Leap



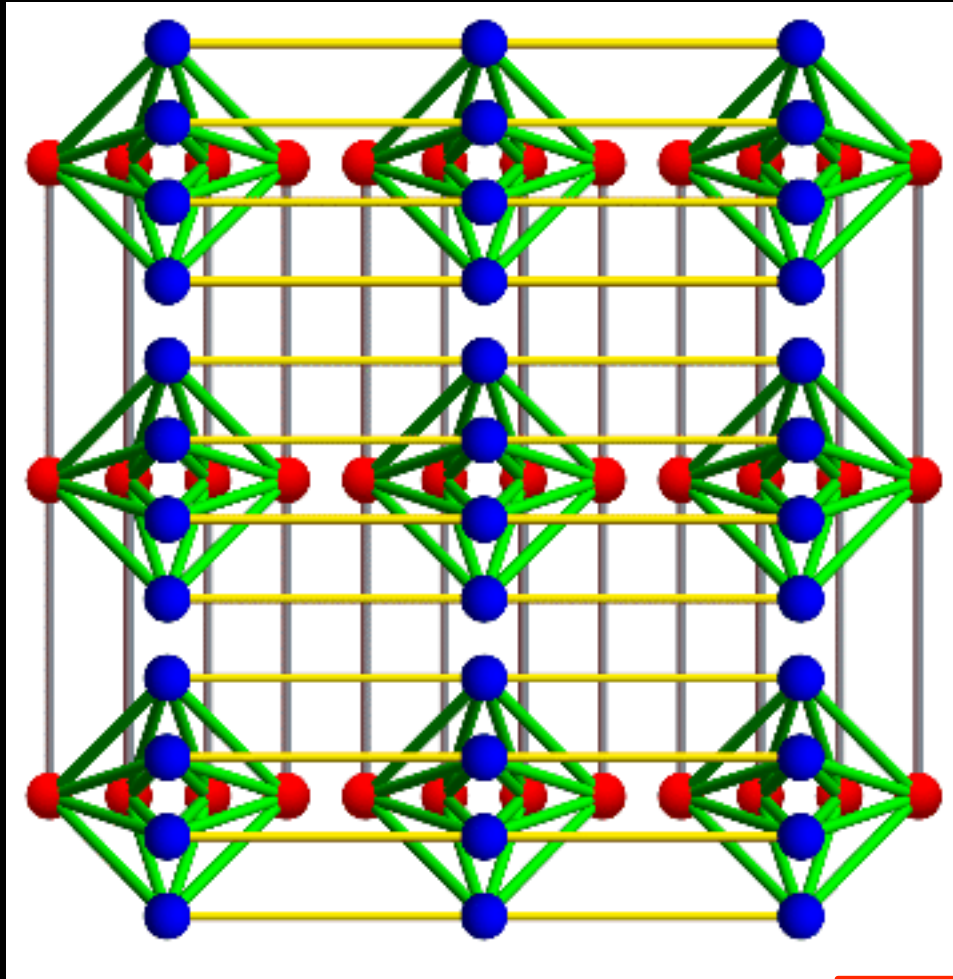
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Take the Leap



# $K_{4,4}$ Chimera



$$3 \times 3$$

$$N = 3 \times 3 \times 8$$

Next an advertisement!!!

Note on redactions.

Redactions are done to:

- Avoid problems with lawyers
- Shield some preliminary data

PHYSICAL REVIEW APPLIED **10**, 054004 (2018)

## **How Small-World Interactions Can Lead to Improved Quantum Annealer Designs**

Helmut G. Katzgraber<sup>1,2,3,4,\*</sup> and M.A. Novotny<sup>5,6,7</sup>

**Without a fully connected graph:**

- **Finite-temperature spin glass transition temperature**
- **Mean-field spin glass critical exponents**

# Ising Spin Glass Scaling

Large scale Monte Carlo simulations to test scaling of Ising spin glasses with SW

$$q = \frac{1}{N} \sum_{j=1}^N S_j^{(\alpha)} S_j^{(\beta)}$$

$$\chi/N = [\langle q^2 \rangle]_{\text{av}} \sim N^{-\Gamma} X(x)$$

$$\Gamma = \frac{\gamma_{\text{BP}} + 2\beta_{\text{BP}}}{d_{\text{upper}} \nu_{\text{MF}}} = \frac{2}{3}$$

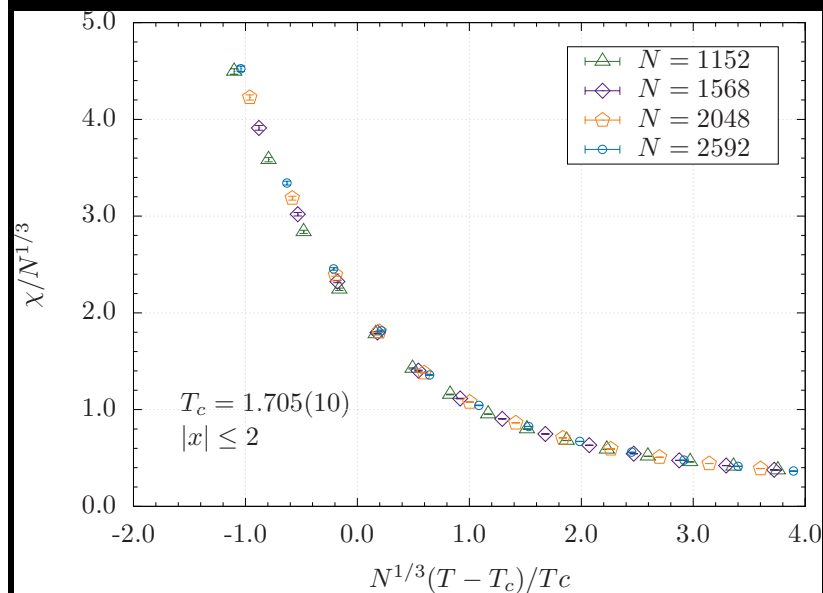
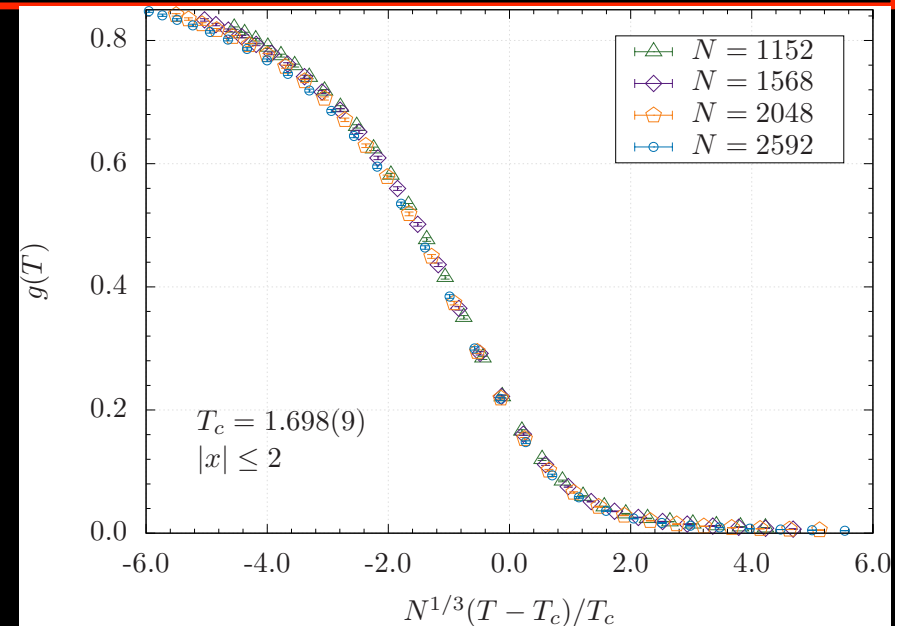
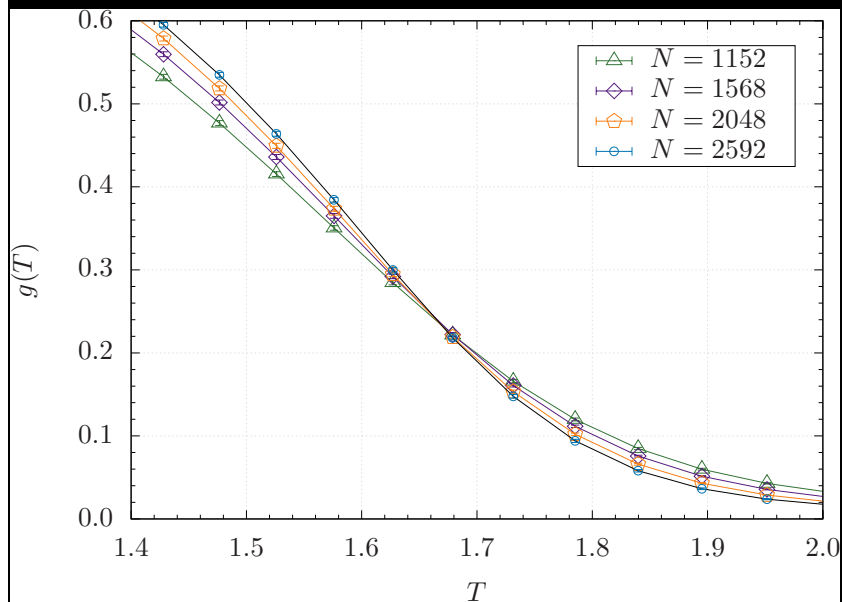
No transition below

$$d_{\text{lower}} \approx 2.5$$

Mean field above

$$d_{\text{upper}} = 6$$

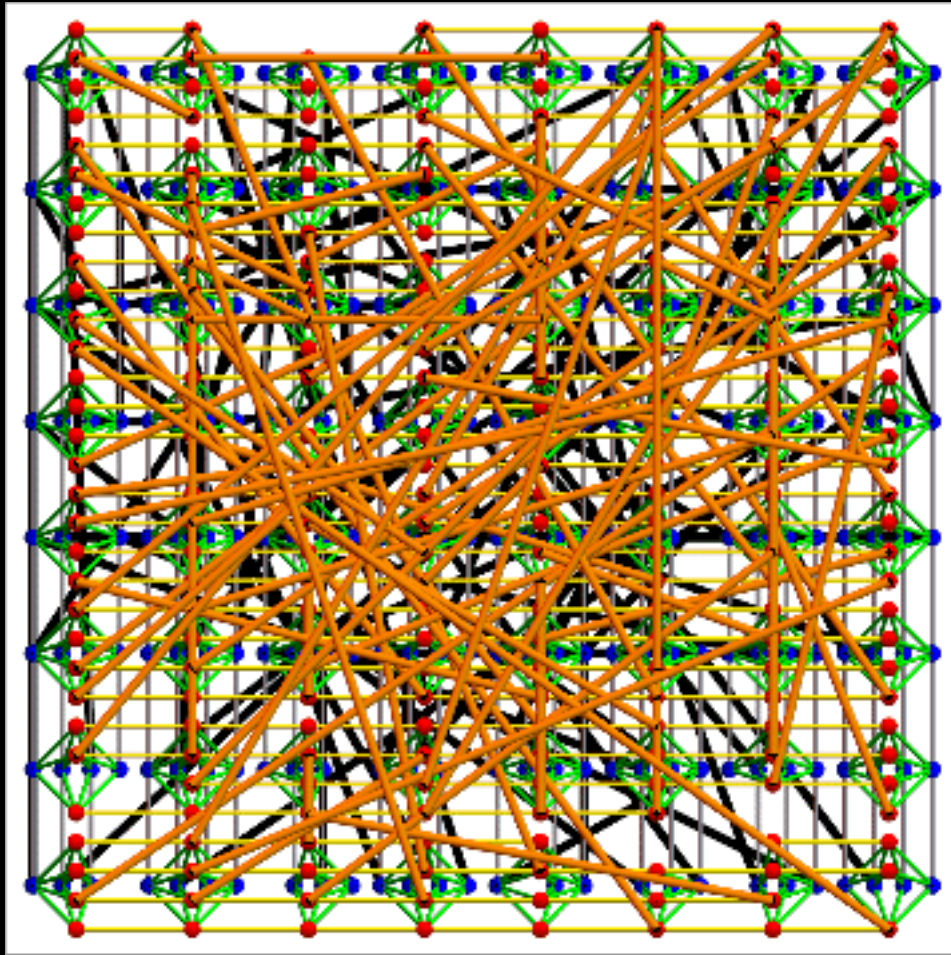
# Small World Ising Spin Glass from Chimera



- Finite  $T_c$
- Mean field exponents



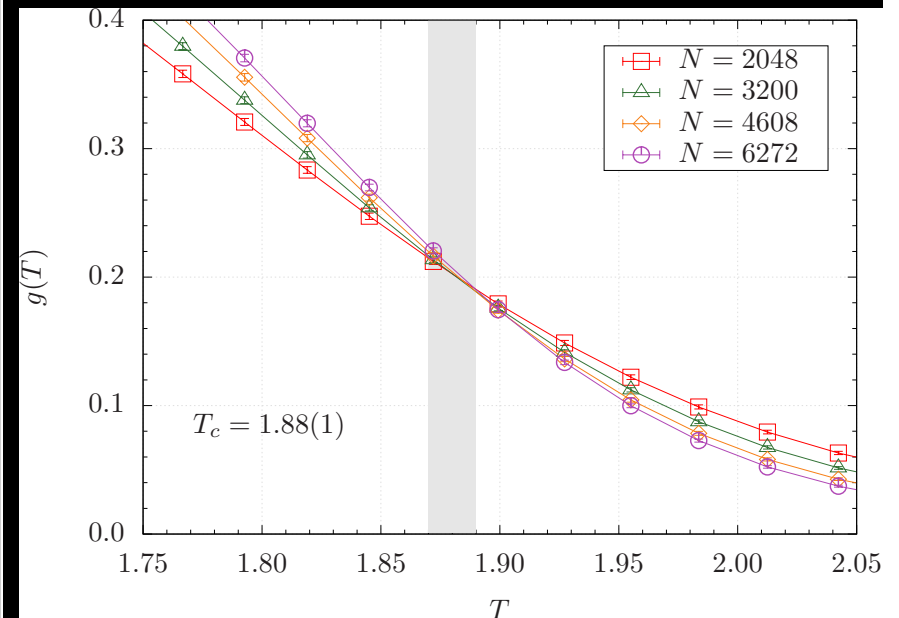
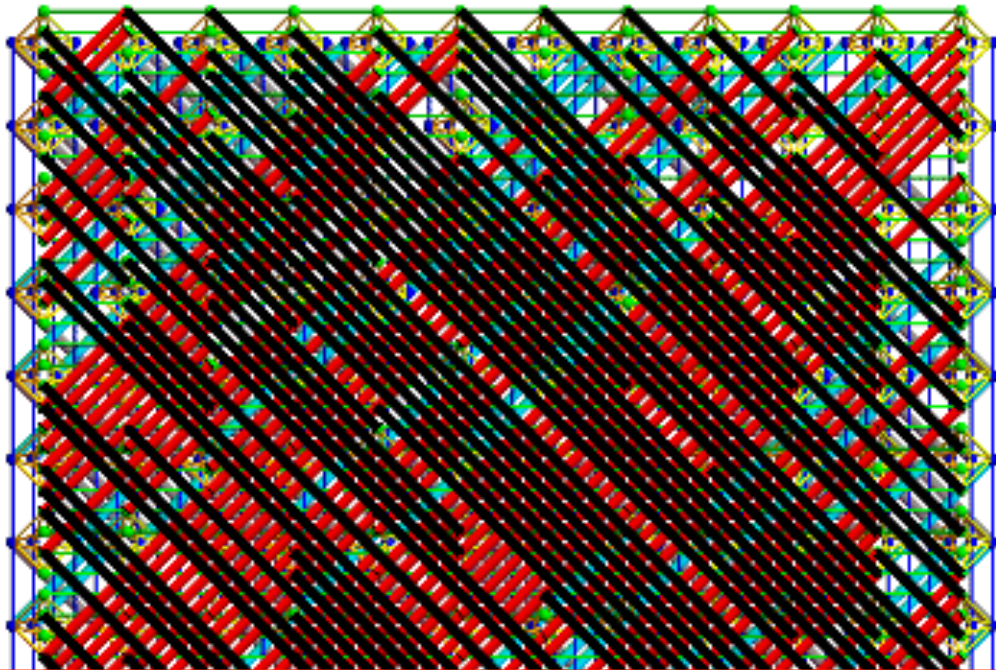
# Small World Ising Spin Glass from Chimera



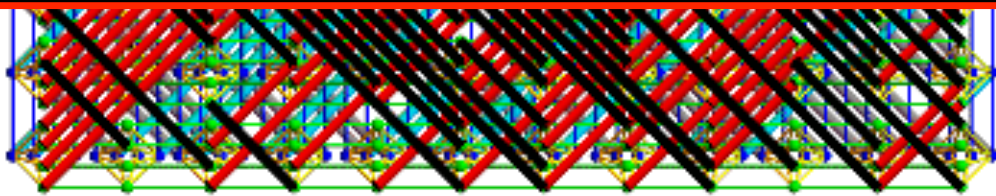
- Finite  $T_c$
- Mean field

**Unbuildable**

# Small World Ising Spin Glass from Chimera Constrained to 4 layers and $\pi/4$



US patent, and US patent pending

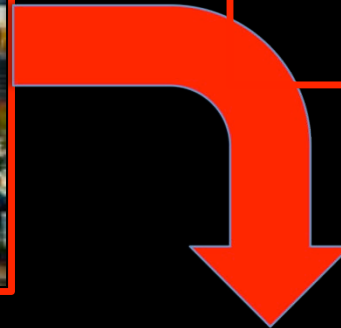


- **Finite  $T_c$**
- **Mean field**

# Forward-Reverse Error Mitigation FREM



Crazy idea at  
'open forum' at  
Qubits 2018

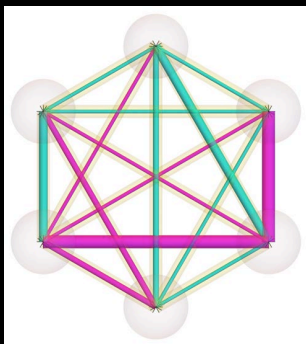


# Forward-Reverse Error Mitigation FREM



- Python package
- Real-time QM solver
- QuTIP 4.4.0
- GitHub availability

Nic Ezzell



- Closed quantum system
- Sidon set  $J_{ij}$  and  $h_j$
- Fully connected graph
- $n$  qubits

# Forward-Reverse Error Mitigation

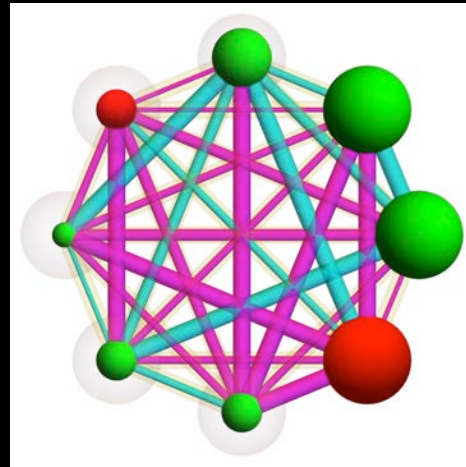
## FREM

Two types of quantum error mitigation:

- **Mathematical / technological**  
*Increase probability finding ground state*
- **Psychological**  
*How much do you trust you have been returned the ground state?*

# *Psychological error in quantum computers?*

Guaranteed to not  
always get the correct  
answer ...



You *give* quantum annealer:  
bias  $h_j$  on each qubit  
coupling  $J_{ij}$  between qubit pairs  
You *get* (hopefully? maybe? probably?)  
ground state spins

# *Psychological error in quantum computers?*

**Guaranteed to not  
always get the correct  
answer ...**

**When a human grossly misattributes the  
probability of success in returning GS**

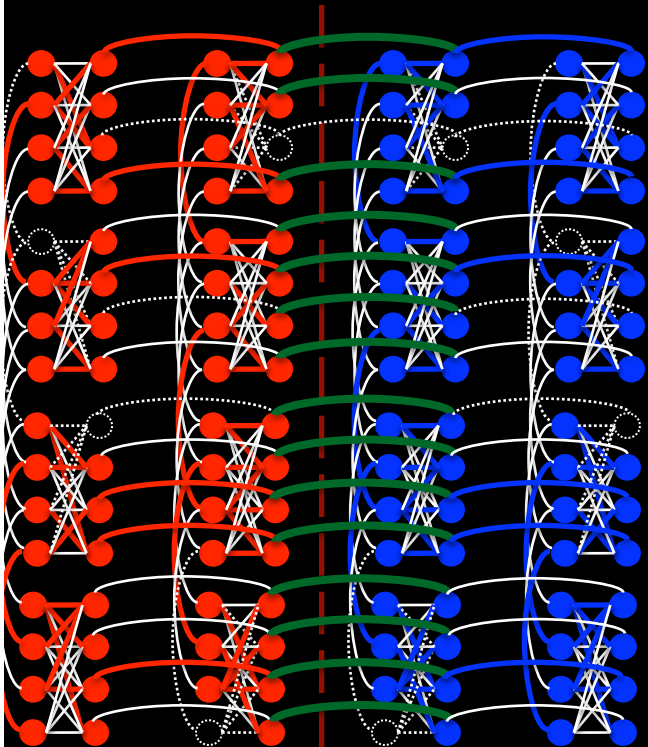
**Ask question in a different way ...**

# Forward-Reverse Error Mitigation FREM

## Previous Error Mitigation

Mizel, et al, 2007

Perera and Novotny 2017 (Answer checking, mirror plane)

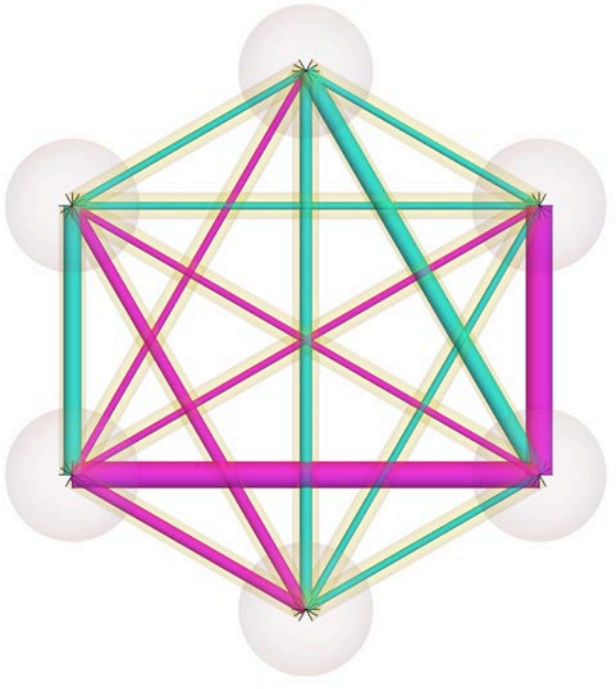


**Both use qubit resources**

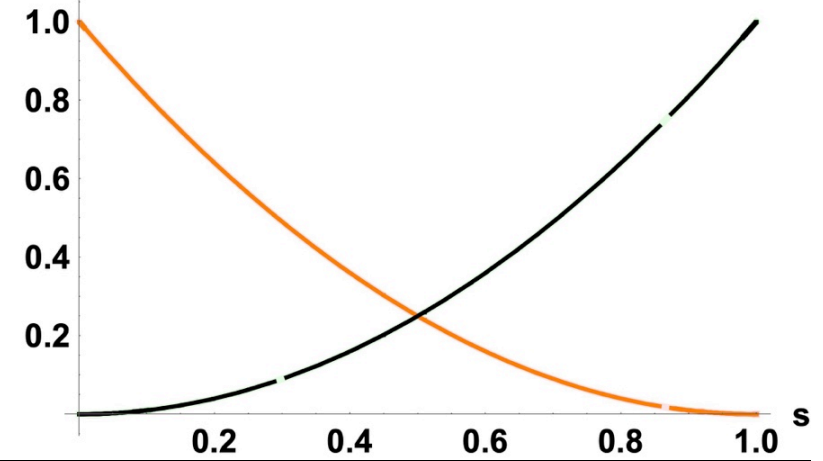
**FREM does not!**



# Forward Annealing

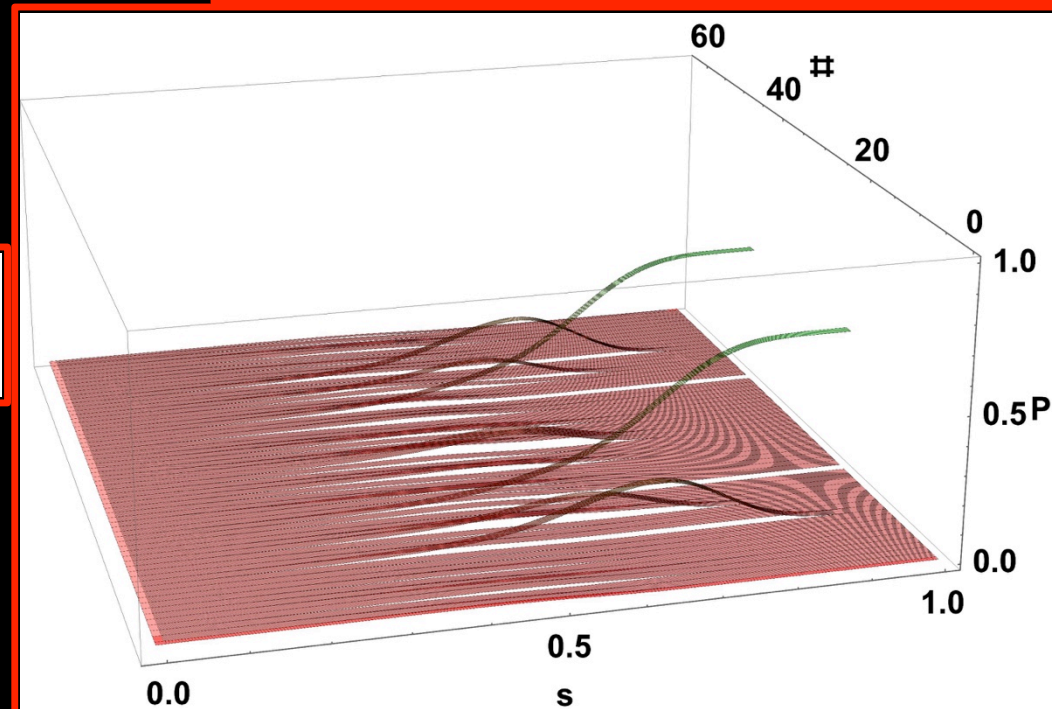


A(s) and B(s)

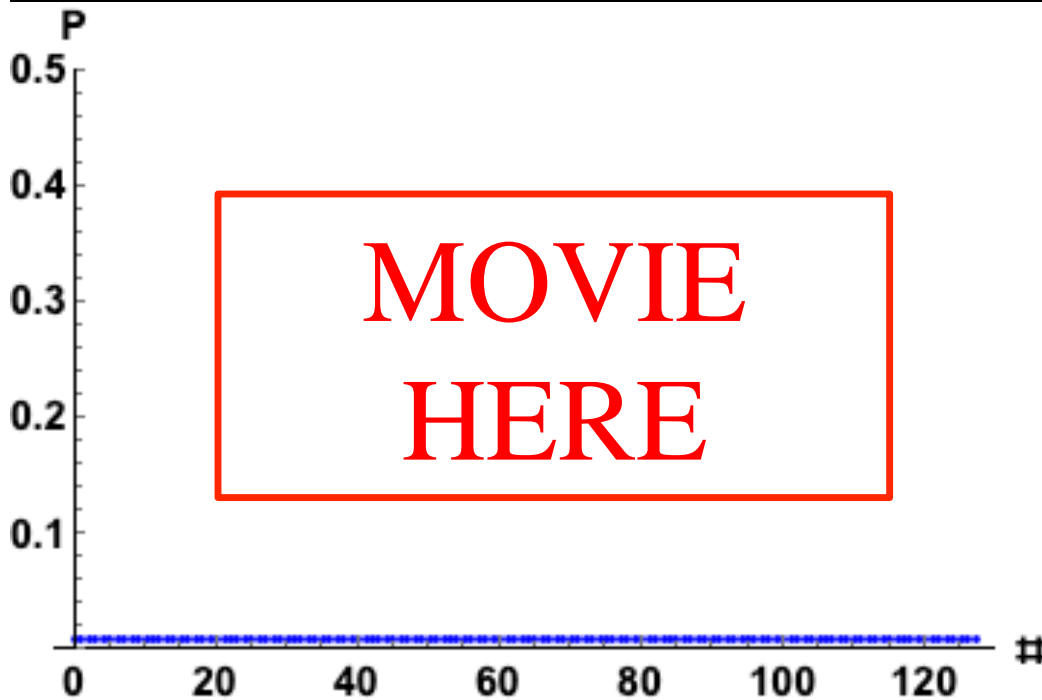


$$\mathcal{H}(s) = A(s)\mathcal{H}_x + B(s)\mathcal{H}_z$$

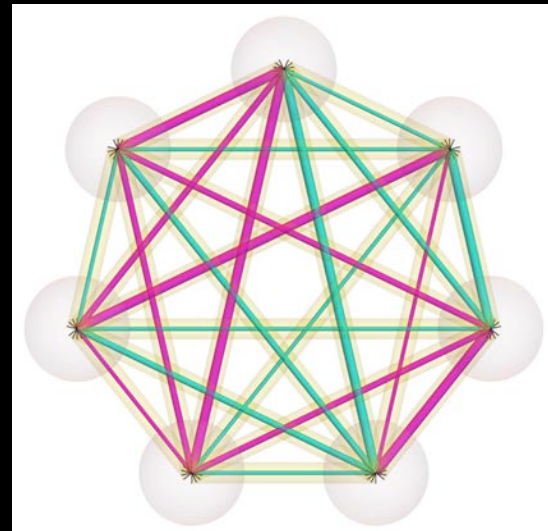
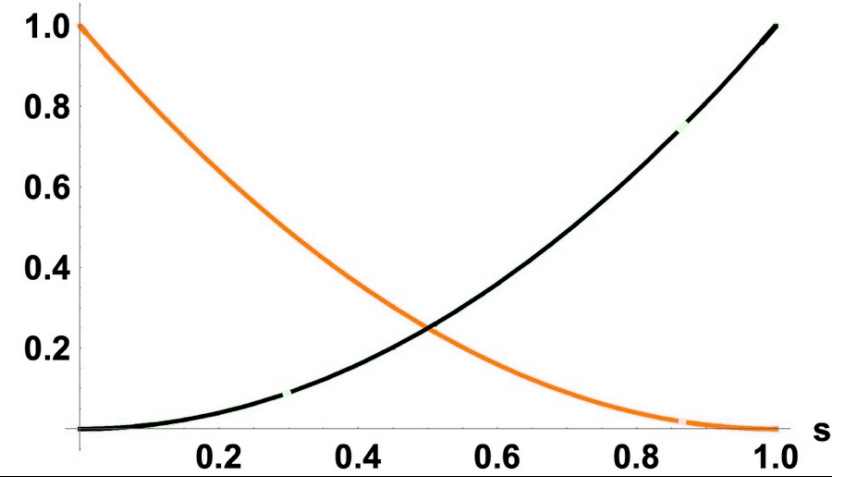
Start in GS of  $\mathcal{H}_x$



# Forward Annealing



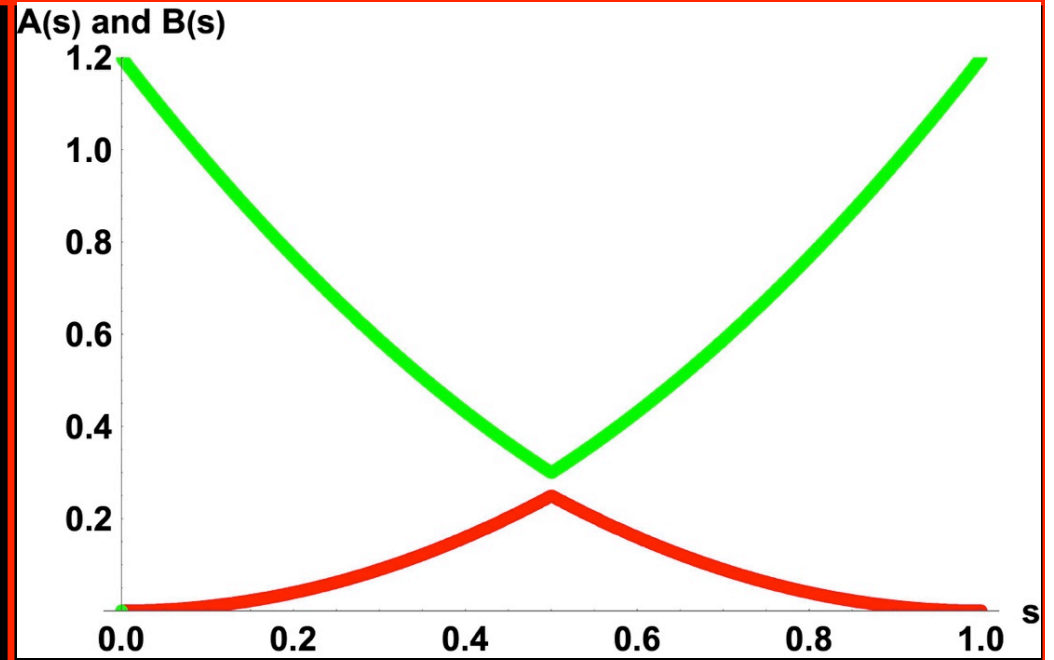
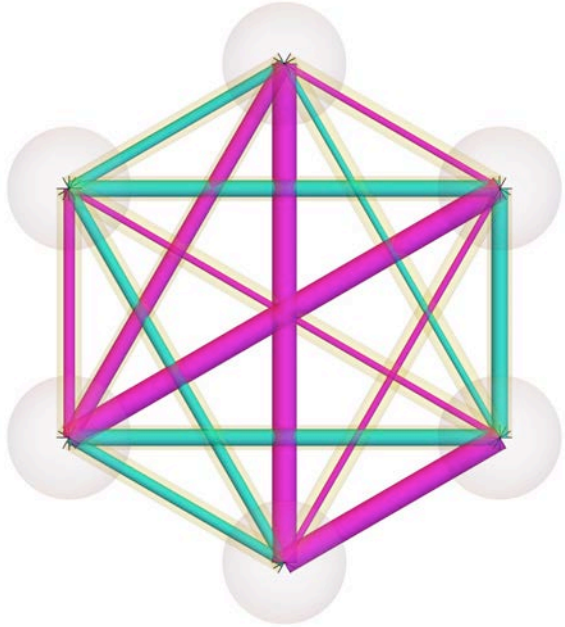
A(s) and B(s)



Start in GS of  $\mathcal{H}_x$

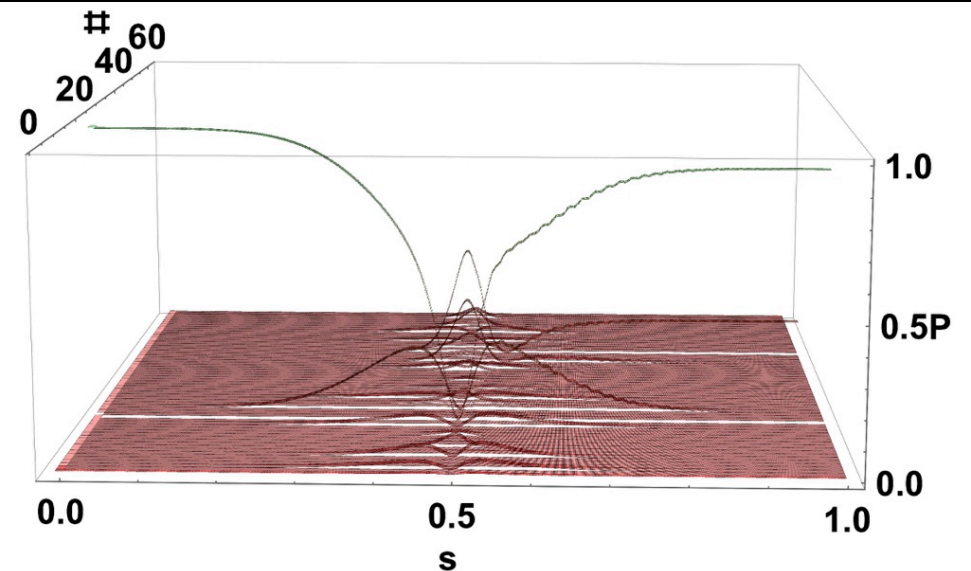
$$\mathcal{H}(s) = A(s)\mathcal{H}_x + B(s)\mathcal{H}_z$$

# Reverse Annealing

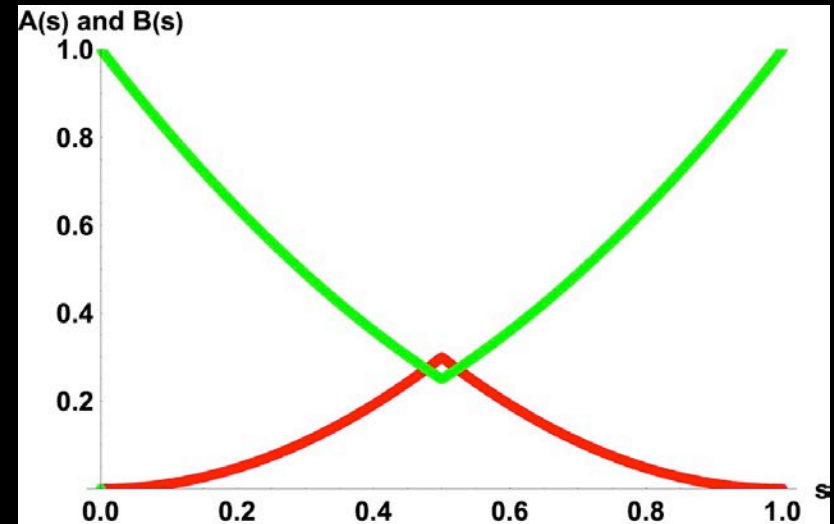
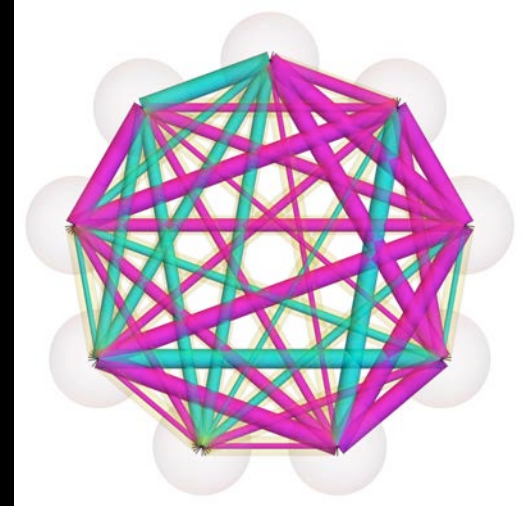


$$\mathcal{H}(s) = A(s)\mathcal{H}_x + B(s)\mathcal{H}_z$$

Start in state of  $\mathcal{H}_z$



# Reverse Annealing



Start in state of  $\mathcal{H}_z$

$$\mathcal{H}(s) = A(s)\mathcal{H}_x + B(s)\mathcal{H}_z$$

# Forward-Reverse Error Mitigation FREM

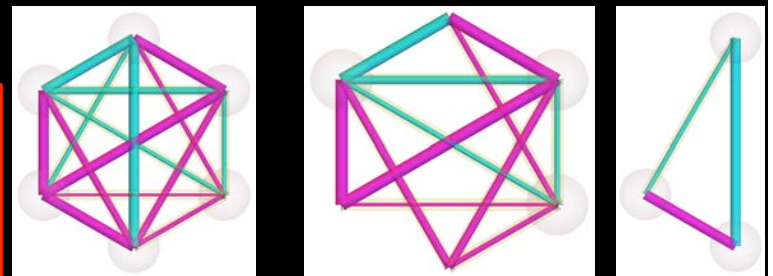
- **F**: Some of Hamiltonian forward annealed
- **R**: Some of Hamiltonian reverse annealed

$$\mathcal{H}(s) = A_F(s)\mathcal{H}_{F,x} + A_R(s)\mathcal{H}_{R,x} + B_F(s)\mathcal{H}_{F,z} + B_R(s)\mathcal{H}_{R,z}$$

$$\begin{aligned}\mathcal{H}_x &= \mathcal{H}_{F,x} + \mathcal{H}_{R,x} \\ \mathcal{H}_z &= \mathcal{H}_{F,z} + \mathcal{H}_{R,z}\end{aligned}$$

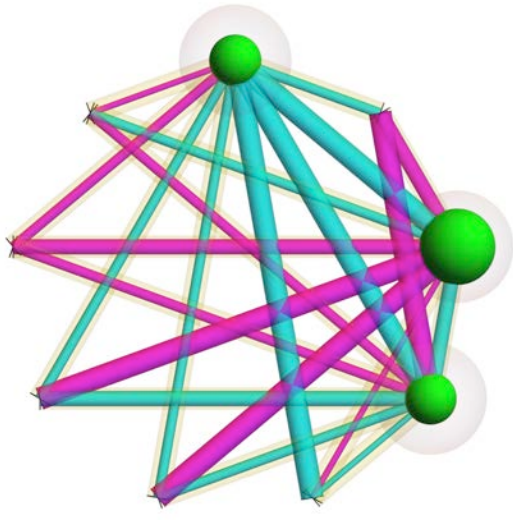
Start in separable GS;

$$\mathcal{H}_{F,x} \boxtimes \mathcal{H}_{R,z}$$

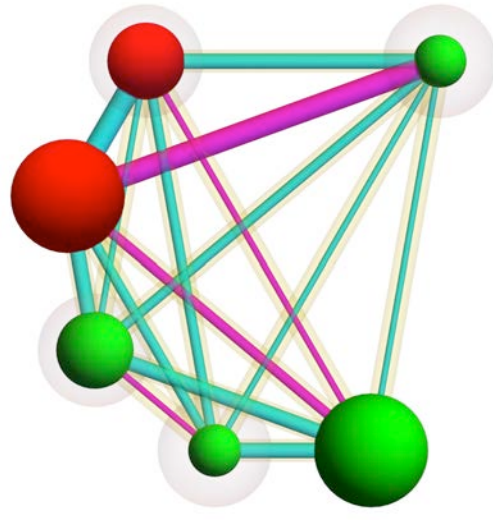


# Forward-Reverse Error Mitigation

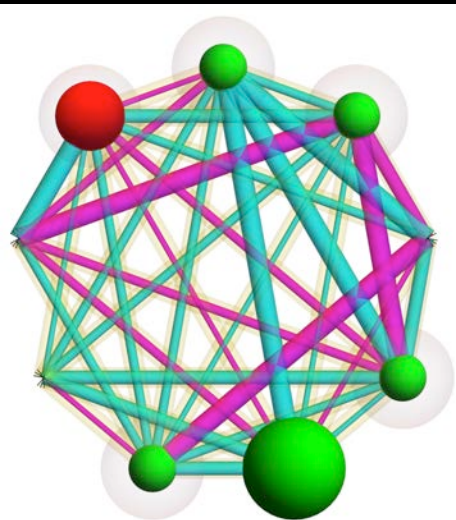
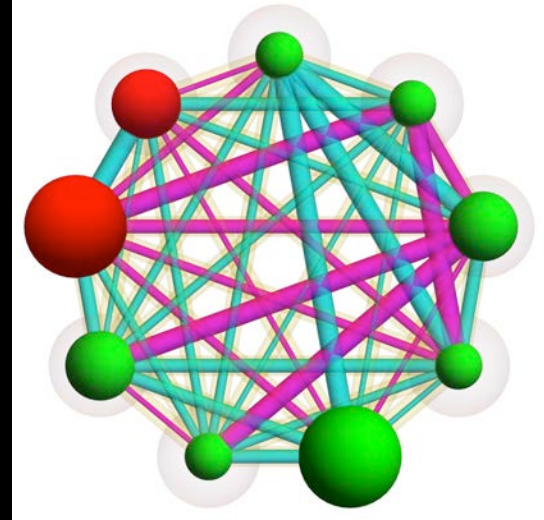
## FREM: Petitioning



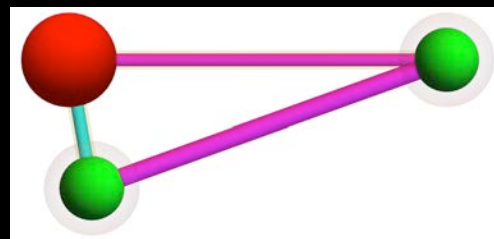
+



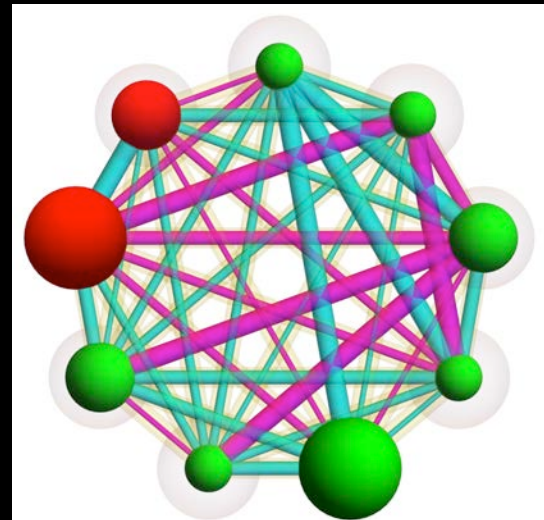
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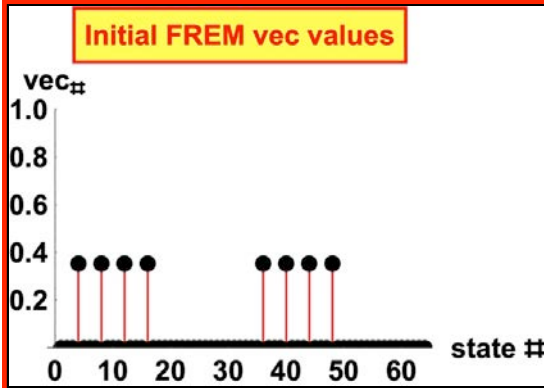
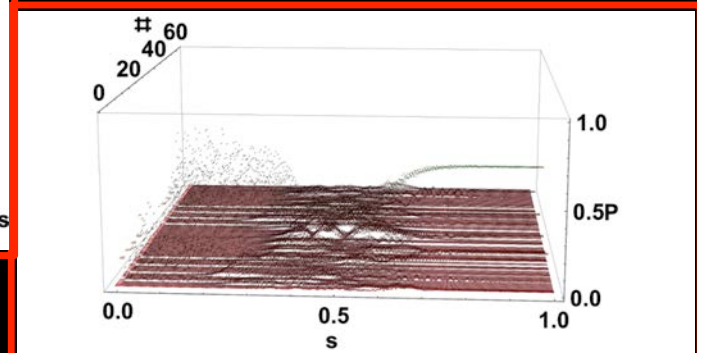
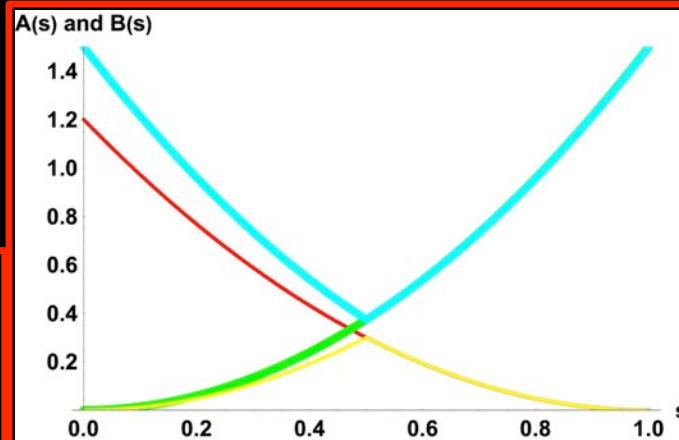


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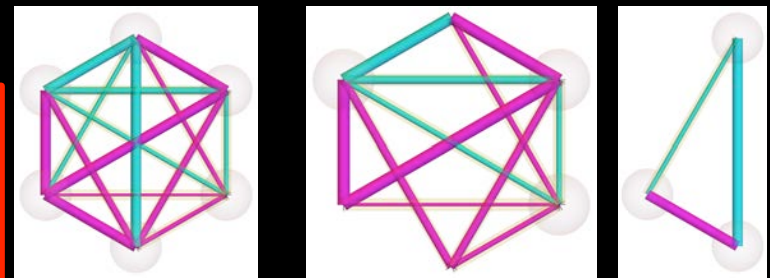
# Forward-Reverse Error Mitigation FREM

$$\mathcal{H}(s) = A_F(s)\mathcal{H}_{F,x} + A_R(s)\mathcal{H}_{R,x} + B_F(s)\mathcal{H}_{F,z} + B_R(s)\mathcal{H}_{R,z}$$



Start in separable GS;

$$\mathcal{H}_{F,x} \otimes \mathcal{H}_{R,z}$$

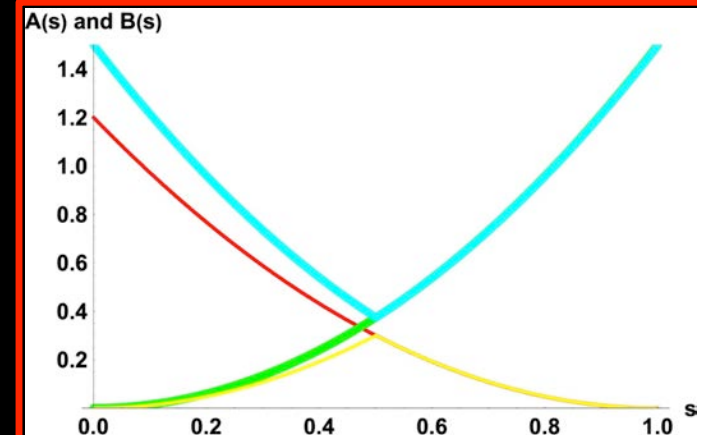


# Forward-Reverse Error Mitigation FREM



Start in separable GS;

$$\mathcal{H}_{F,x} \boxtimes \mathcal{H}_{R,z}$$





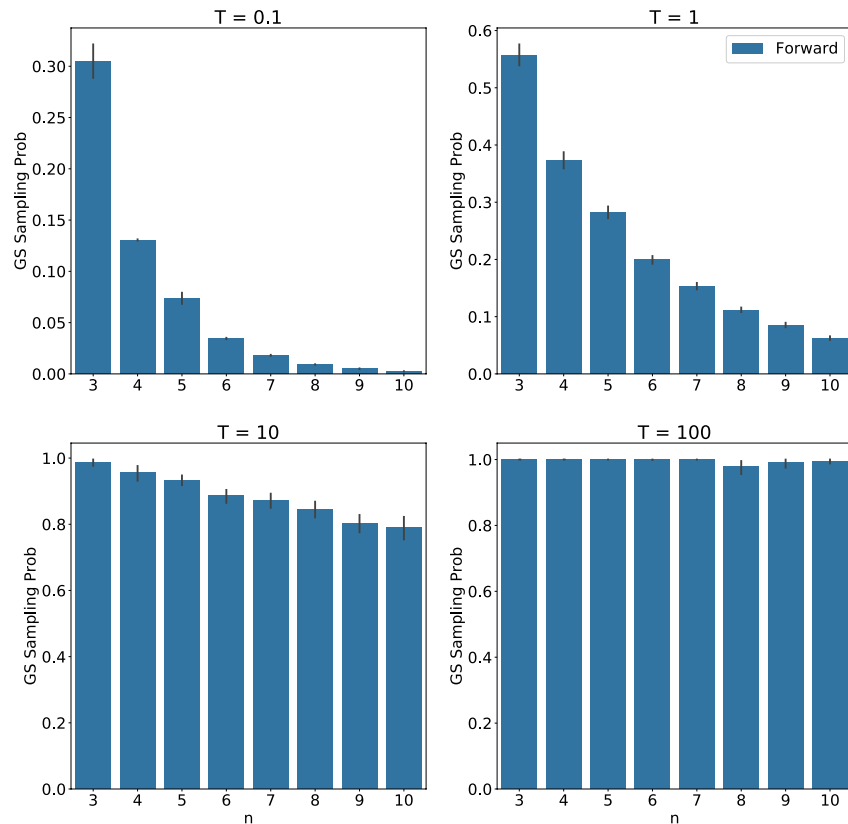
**Forward anneal?**  
**Reverse anneal?**  
**FREM?**

**Adiabatic theorem:**  
**Start in ground state**  
**End anneal in ground state superposition**

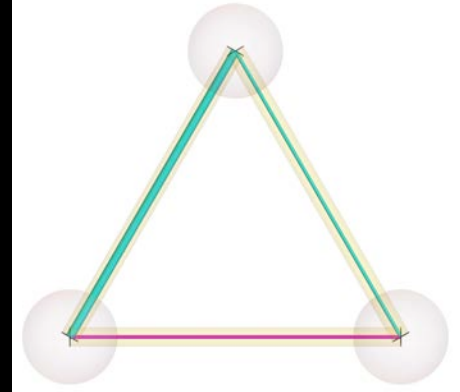
Theorist claims quantum supremacy achieved in 2009!

- If only company had achieved  $T=0$
- Demonstration ongoing to achieve  $t \rightarrow \infty$

# Forward Annealing test:

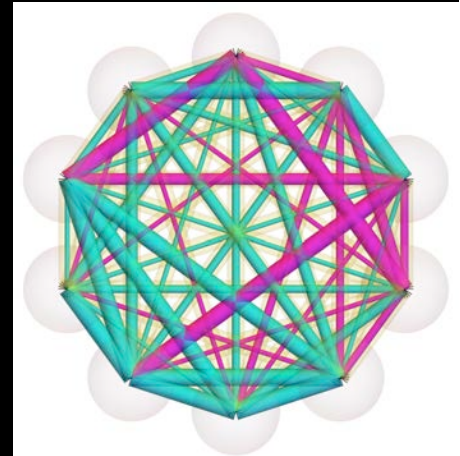


$$n = 3$$



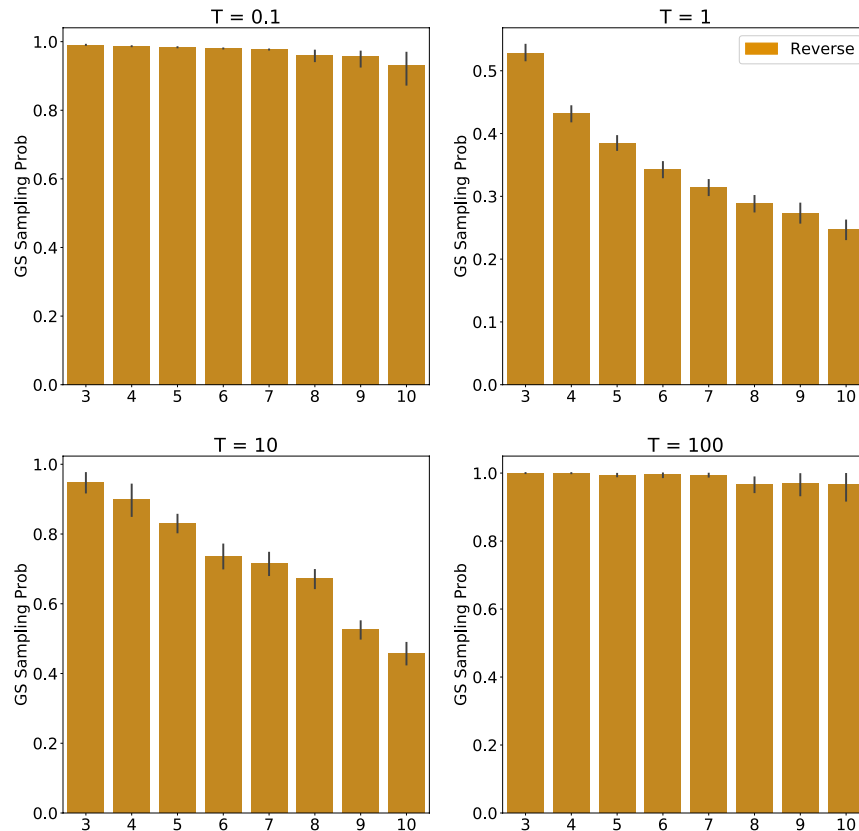
Anneal times  $T$   
 $n = 3$  thru  $10$

$$n = 10$$

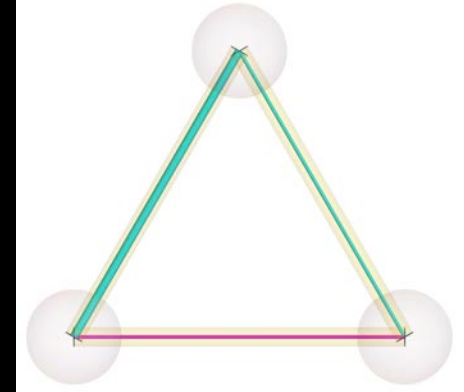


Start in GS of  $\mathcal{H}_x$

# Reverse annealing test to $s=0.5$ :

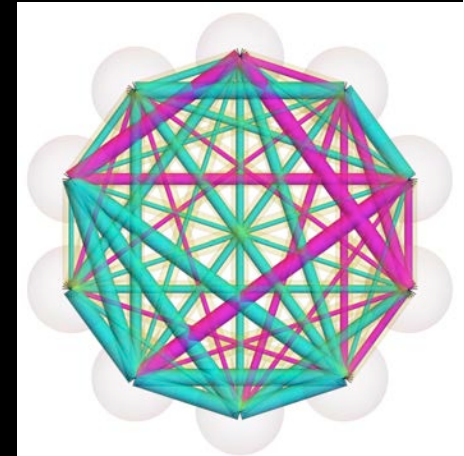


$$n = 3$$



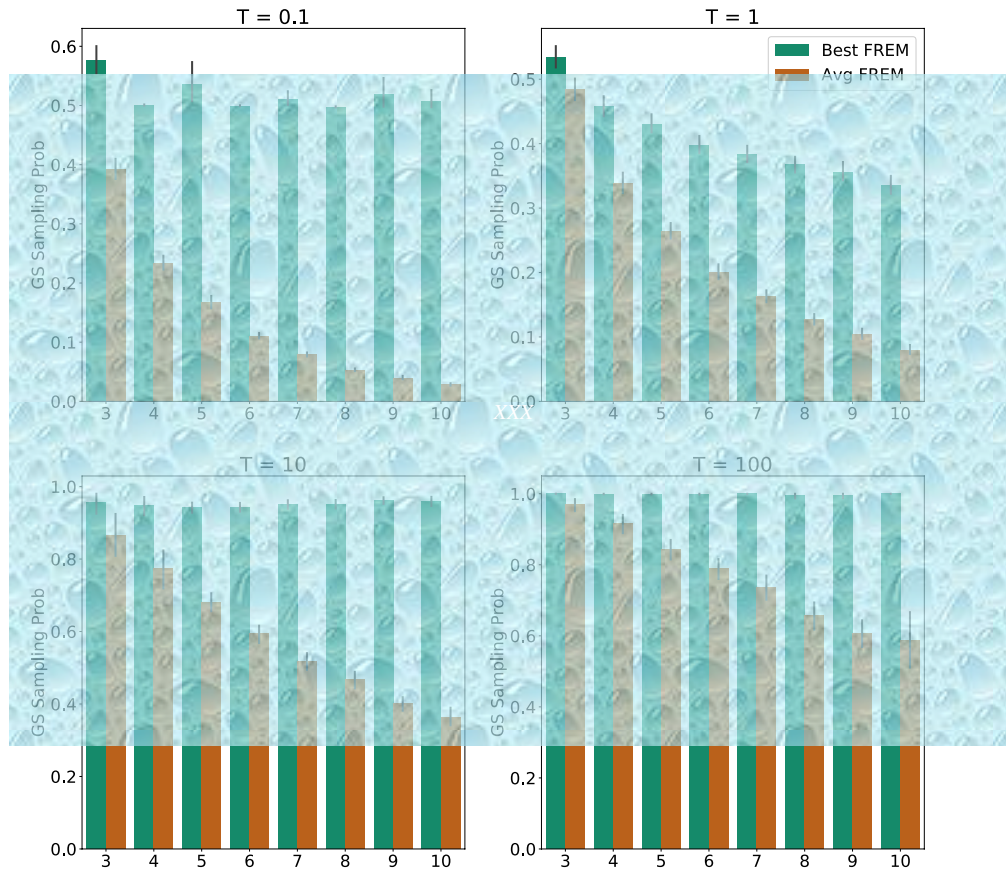
Anneal times  $T$   
 $n = 3$  thru  $10$

$$n = 10$$

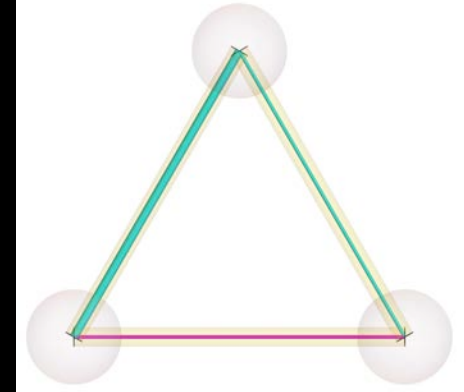


Start in GS of  $\mathcal{H}_z$   
from forward anneal

# FREM test to $s=0.5$ :

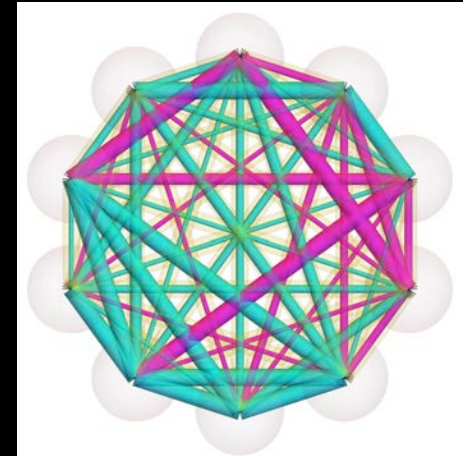


$n = 3$



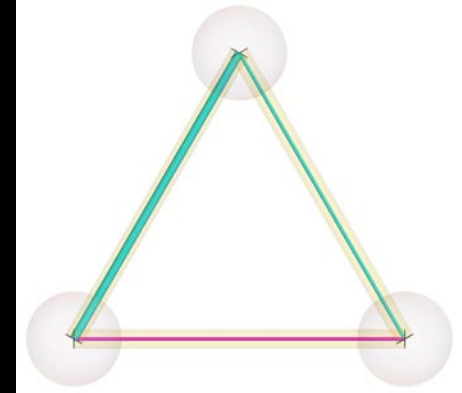
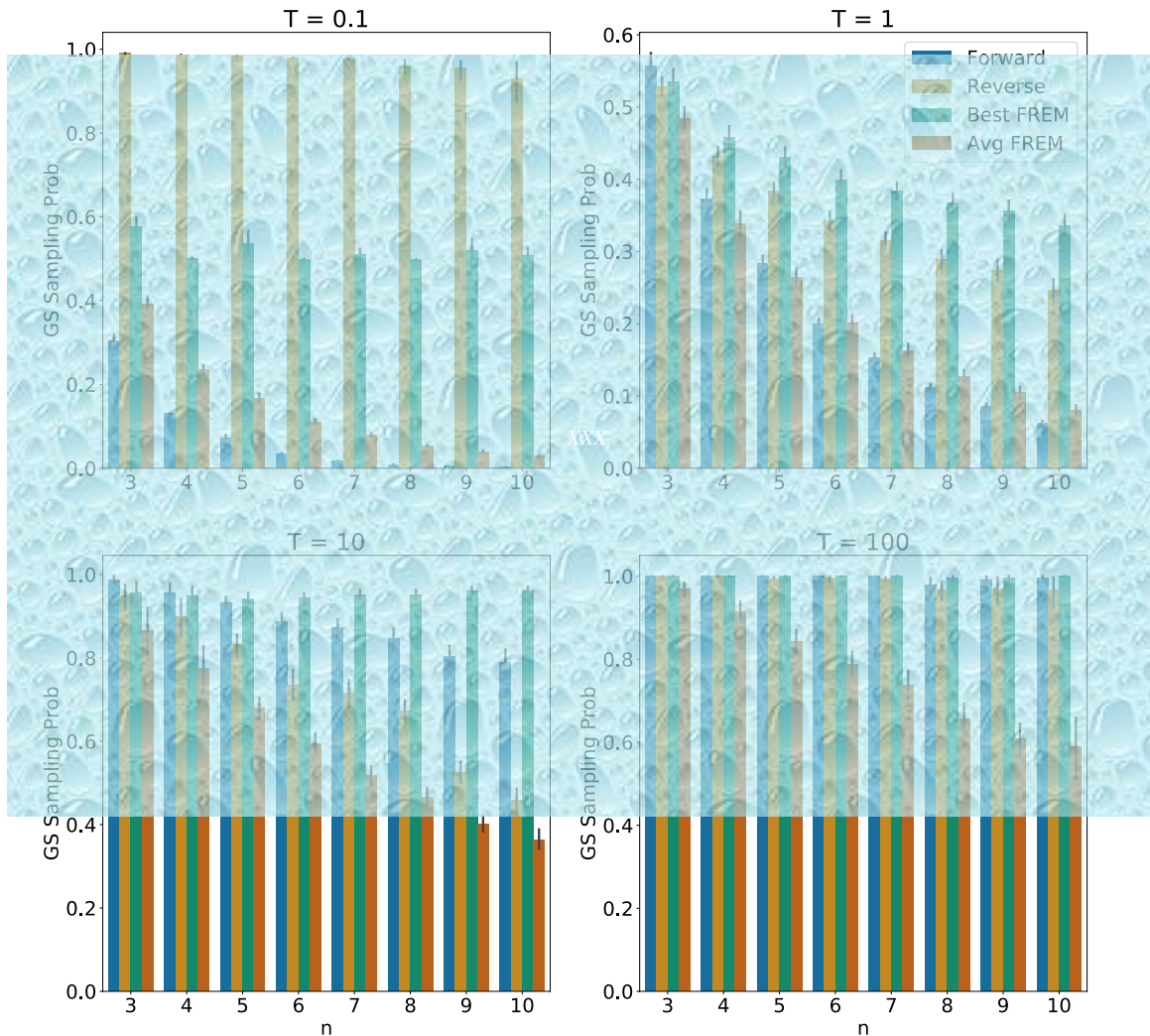
Anneal times  $T$   
 $n = 3$  thru  $10$

$n = 10$

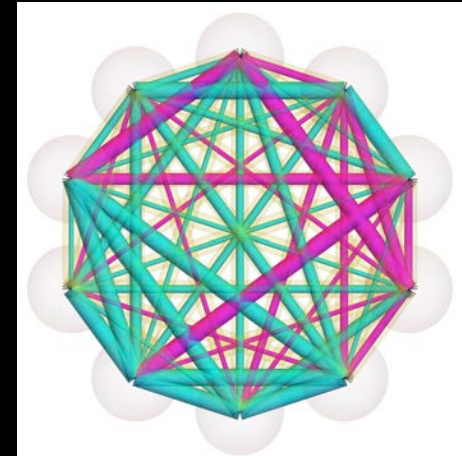


Best and all FREM partitioning

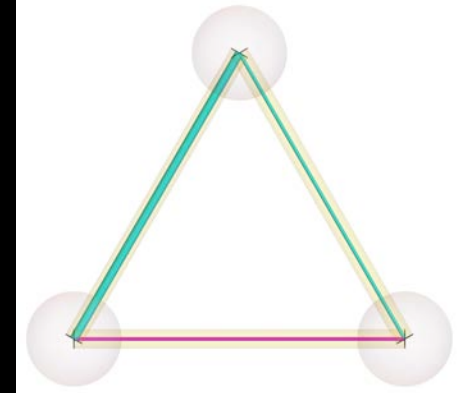
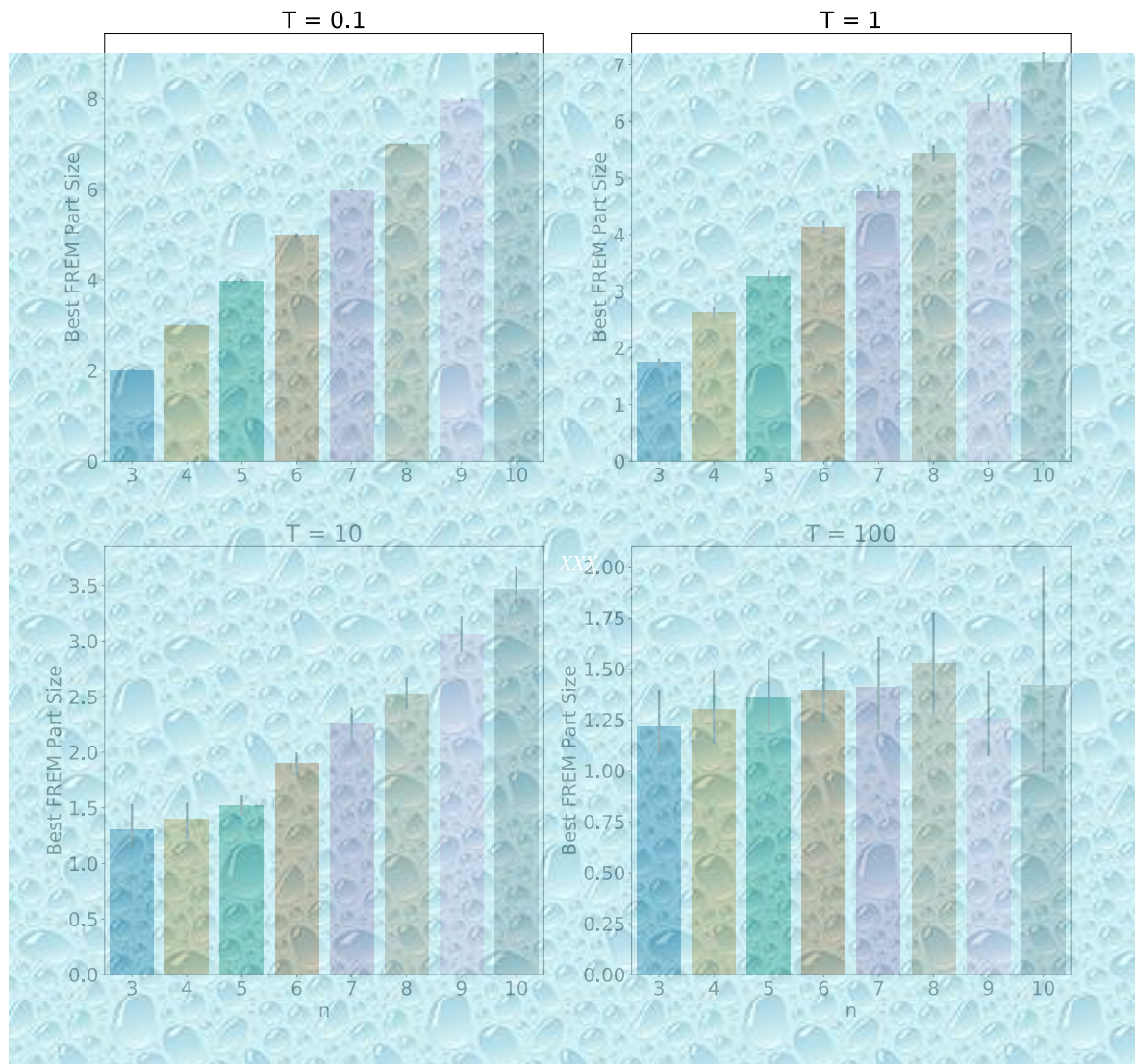
# Compare: FREM, Forward, Reverse



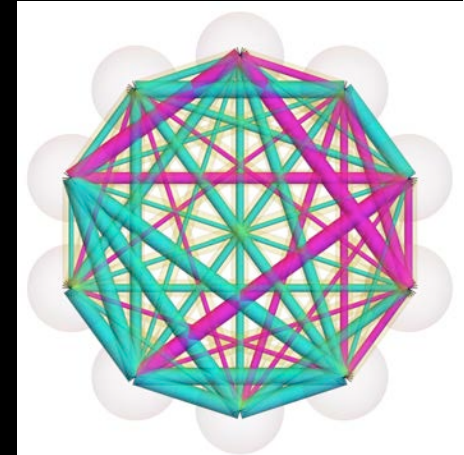
Anneal times  $T$   
 $n = 3$  thru  $10$



# FREM: best partition size



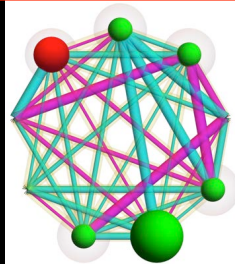
Anneal times  $T$   
 $n = 3$  thru  $10$



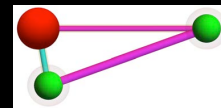
# Conclusions and Discussion



- ❖ Use D-Wave unique resources
- ❖ Teach quantum computing and D-Wave
- ❖ Work to improve next generation machines
- ❖ Graph: If you cannot build complete graph a small-world graph should be your next option
- ❖ FREM: forward & reverse anneal  
Some partitions increase success probability  
Can overcome ‘psychological errors’



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