

## Route Optimization for Multimodal Transport Systems

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#### **DENSO Future direction**



DENSO Integrated Report 20 Cover Story

#### Crafting the Core— Crafting a new core with our technologies in anticipation of change

Due to companies from other industries entering the market and fierce technological competition, the automotive industry is currently approaching a paradigm shift, which is said to occur once every 100 years. Fully understanding the wave of changes that it faces, DENSO will clear the way for a new motorized society by enhancing and evolving its technologies.



#### **DENSO Efficient Driving**

DENSO envisions a future in which mobility is more efficient and driving is more fun. We are developing electrified technologies for a wide range of vehicles, from gasoline and diesel vehicles to HEV, PHEV, EV and FCV, to improve efficiency with better management of electric, kinetic and heat power. By predicting road conditions and charting the best course, our goal is to reduce energy loss, so people can drive as they wish while also being environmentally friendly.



DENSO Integrated Report 201 Cover Story



#### **DENSO** Connected Driving

DENSO envisions a future in which mobility is connected inside and outside of the vehicle, including cars, people and infrastructure, as well as new services. It brings us new experiences for traveling, and helps us develop automated driving systems that are more convenient and comfortable yet extremely energy efficient. Of course, security issues have emerged from connectivity, such as hackers and data leaks, but with an unwavering focus on safety, DENSO will help protect people and cars.





#### DENSO Automated Driving

DENSO envisions a future in which everyone can travel freely and safely, regardless of their age or physical condition. That's why DENSO is deeply focused on advances in safety and security. Our goal is to evolve our sensing, information  $\delta$ communication and Al technologies to eliminate limitations to mobility.

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Qubits Europe 2019 / 26th March 2019 / Akira Miki © DENSO CORPORATION All Rights Reserved. DENSO X QUANTUM

#### **Team & projects**





## **Motivation**



# Sharing economy is one of the market with growth potential



# Sharing economy with transport system like Uber becomes





https://www.uber.com/ride/express-pool/

We would propose optimization of advanced transport system with sharing





#### **System overview**







2. Route optimization of multimodal transport system





## **Optimization of multi modal sharing**







2. Each small vehicle like taxi makes a route for customers and an optimized meeting point (junction) of large vehicle (VRP)



 $\mathbf{J}_{3}$ 

 $J_4$ 

K Customers

destinations

4. Each small vehicle make a route from an optimized junction and customer destinations (VRP)

 $\mathbf{J}_{2}$ 

**DENSO** Crafting the Core Customers'

departures

## Our formulation of the optimization



 Conventional method (VRP iterations: linear programming (LP))



Optimized by linear programming solver
Gurobi

Proposed method (QUBO / quadratic programming (QP))

Minimizing cost from all combinations of junctions



- Optimized by quadratic programming solver
  - Gurobi
- Ising model hardware
  - D-Wave

#### Benchmarking D-Wave with proprietary solver Gurobi



#### **Decision Variables for QUBO formulation**



• x variables for carts:  $x_{g,t,c}^{(1)}$ : pick up,  $x_{g,t,c}^{(2)}$ : drop off

 $x_{g,t,c}^{(1)} = \begin{bmatrix} 1: \text{ pick up / drop off guest g at time t for cart c} \\ 0: \text{ not pick up / drop off guest g at time t for cart c} \end{bmatrix}$ 

• y variables for shuttle:  $y_i^{(1)}$ : pick up,  $y_i^{(2)}$ : drop off

 $y_j^{(1)} = \begin{bmatrix} 1: \text{ stop j junction for picking up / dropping off guests} \\ 0: \text{ not stop j junction for picking up / dropping off guests} \end{bmatrix}$ 



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#### **Objectives for QUBO formulation (1/6)**



 $L_1 = \sum_{c=1}^C \sum_{g=1}^G \mathscr{C}_{j_g,g} x_{g,1,c}^{(1)} \qquad j_g: \text{ nearest junction from guest } g$ 

Cost (distance) from start position of cart to first guest position





#### **Objectives for QUBO formulation (2/6)**



$$L_{2} = \sum_{c=1}^{C} \sum_{t=1}^{T-1} \sum_{g_{2}=1}^{G} \sum_{g_{1}=1}^{G} \ell_{g_{1},g_{2}} x_{g_{1},t,c}^{(1)} x_{g_{2},t+1,c}^{(1)} + \sum_{c=1}^{C} \sum_{j=1}^{J} \sum_{g=1}^{G} \ell_{g,j} x_{g,T,c}^{(1)} y_{j}^{(1)}$$

Cost (distance) from first guest position to last guest position and from last guest position to junction of shuttle coming





## **Objectives for QUBO formulation (3/6)**



$$L_3 = \sum_{j=1}^{J} \ell_{j_s,j} y_j^{(1)} \qquad j_s: \text{ junction of initial shuttle position}$$

Cost (distance) from start position of shuttle to junction of cart coming





#### **Objectives for QUBO formulation (4/6)**



$$L_4 = \sum_{j_1=1}^J \sum_{j_2=1}^J \mathscr{C}_{j_1, j_2} y_{j_1}^{(1)} y_{j_2}^{(2)}$$

Cost (distance) between junctions of shuttle running





#### **Objectives for QUBO formulation (5/6)**



$$L_{5} = \sum_{c=1}^{C} \sum_{j=1}^{J} \sum_{g=1}^{G} \ell_{j,g} y_{j}^{(2)} x_{g,1,c}^{(2)} + \sum_{c=1}^{C} \sum_{t=1}^{T-1} \sum_{g_{2}=1}^{G} \sum_{g_{1}=1}^{G} \ell_{g_{1},g_{2}} x_{g_{1},t,c}^{(2)} x_{g_{2},t+1,c}^{(2)}$$

Cost (distance) from junction to first guest position and from first guest position to last guest position





#### **Objectives for QUBO formulation (6/6)**



 $L_6 = \sum_{c=1}^C \sum_{g=1}^G \mathscr{C}_{j_g,g} x_{g,T,c}^{(2)} \quad j_g: \text{ nearest junction from guest } g$ 

Cost (distance) from last guest position to end position of cart





#### **Constraints for QUBO formulation (1/3)**



Constraint rule 1: each time and cart have each one guest pick up or drop off by cart





#### **Constraints for QUBO formulation (2/3)**



Constraint rule 2: each guest has each one condition of pick up or drop off







#### **Constraints for QUBO formulation (3/3)**



Constraint rule 3: shuttle has each one junction stop for pick up or drop off



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#### **QUBO** formulation and problem settings





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## **Embedded** *h*<sub>0</sub> normalization



Without  $h_0$  normalization

$$H = \sum_{i=1}^{N} \sum_{j=1}^{N} Q_{ij} x_i x_j$$
$$Q_{ij} = 4 * Q_{ij} / max(abs(Q_{ij}))$$

With  $h_0$  normalization  $H = \sum_{i=1}^{N} \sum_{j=1}^{N} Q_{ij} x_i x_j$   $Q_{ij} = 4 * Q_{ij} / max(abs(Q_{ij}))$   $h_{0_i} = 2 * h_{0_i} / max(abs(h_i))$   $h_0: e$ 

 $j_{0_{ij}} = 1 * j_{0_{ij}} / max(abs(J_{0_{ij}})) \quad j_0: \epsilon$ 

 $h_0$ : embedded linear Ising coefficients.  $j_0$ : embedded quadratic Ising coefficients

Num. of guests = 2, Num. of variables = 28

	Without h <sub>o</sub> ı	normalization	With h <sub>0</sub> normalization		
	auto_scale=True	auto_scale=False	auto_scale=True	auto_scale=False	
Percentage of exact [%]	$0.95 \pm 0.78$	$0.95 \pm 0.47$	29.9 ± 10.3	27.1 ± 8.95	
Percentage of valid [%]	74.8 ± 6.74	64.73 ± 9.95	42.5 ± 11.9	$40.3 \pm 10.8$	

Num. of guests = 4, Num. of variables = 52, Exact:  $0.69 \pm 0.52$  (without), Exact: 0.0 (with) Num. of guests = 6, Num. of variables = 92, Exact: ~ 0.003 (without), Exact: 0.0 (with)

#### For small-case (28 variables), embedded $h_0$ normalization helps to adjust parameters for D-Wave



#### **Results of D-Wave and Gurobi**

Num. of guests = 2, Num. of v	variables = 28		
	D-Wave	Gurobi	
	Quadratic	Quadratic	Linear
Run time [sec]	7.6 ± 3.9 (internal 1.6)	$0.02 \pm 0.002$	$0.31 \pm 0.03$
Num. of solutions	10000	1	1
Percentage of exact solution [%]	29.9	0	100
Best solution	4.289	4.306 (+0.4%) 🚏	4.289
Time to solution [sec]	0.0099 (internal 0.0021) 🏺	-	0.31



 $TTS(p) = t_c \frac{log(1-p)}{log(1-P_0)} \qquad \begin{array}{l} p = 0.99\\ t_c = run \text{ time / num. of sol.}\\ P_0 = \text{Percentage of exact sol.} \end{array}$ 

- Gurobi (QP) is faster than Gurobi (LP) but not reached exact solution
- D-Wave is faster than Gurobi on time to solution under 52 variables due to exact percentage decrease (next slide)

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## **D-Wave exact solution details**

Num. of guests = 2, Num. of variables = 28



API	dwave_sapi2					Ocean				
Virtual Graph	False					False		True		
Post- process	None	Sampli ng	Optimiz	ation		Optimiz	zation		H	elp me!
Broken Chain	Minimize Energy		Vote	Weighted Random	Vote	Weighted Random	Vote	Weighted Random	Minimize Energy	
Exact [%]	9.8 ± 2.1	0.0	29.9 ± 10.3	0.0	1.06 ± 0.33	0.0	0.0	0.0	0.0	Not work
Valid [%]	43.5 ± 5.68	7.28 ± 0.24	42.5 ± 11.9	0.0	9.8 ± 1.5	0.30 ± 0.10	7.91 ± 12.3	0.044 ± 0.07	0.00071 ± 0.00063	Not work

Num. of guests = 4, Num. of variables = 52, Exact:  $0.69 \pm 0.52$ , Valid:  $35 \pm 9.7$  (opt. & min. energy) Num. of guests = 6, Num. of variables = 92, Exact: ~ 0.003, Valid:  $1.8 \pm 0.34$  (opt. & min. energy)

- Post-process and broken chain works well for the problem but virtual graph not work efficiently
- For large-cases (> 28 variables), the way to improve exact percentage become more important



#### **D-Wave feasible solution details**





- Time to feasible solution increase dependence on variables suppressed compared to time to solution (suppressed percentage decrease)
- Most feasible solutions of D-Wave were within 10%
- Embedding would be an issue to get better results



## Summary



- 1. We propose QUBO formulation of multi modal transportation with selective junctions of small vehicles and large vehicles
- 2. Until 6 guests, exact solutions found by D-Wave
- 3. For small-case (2 guests), embedded  $h_0$  normalization helps to increase exact solutions
- 4. For small-cases (2 guests), D-Wave is faster than Gurobi on time to solution, but large-case (>2 guests) slower due to exact solution decrease
- 5. Most feasible solutions of D-Wave were within 10%

#### **Future work**

- 1. Optimizing real examples of multi modal transportation
- 2. Benchmarking (Digital Ising machines, heuristic algorithms)
- 3. Better embedding methods for improving large-case problems



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## **VRP formulation for linear programming**



min	$\sum_{i=0}^{n+1} \sum_{j=0}^{n+1} c_{ij} x_{ij}$		(2.1)
s.t.	$\sum_{\substack{j=1\\i\neq i}}^{n+1} x_{ij} = 1,$	$i=1,\ldots,n,$	(2.2)
	$\sum_{\substack{i=0\\i\neq h}}^{n} x_{ih} - \sum_{\substack{j=1\\j\neq h}}^{n+1} x_{hj} = 0,$	$h=1,\ldots,n,$	(2.3)
	$\sum_{j=1}^{n} x_{0j} \le K,$		(2.4)
	$y_j \ge y_i + q_j x_{ij} - Q(1 - x_{ij}),$	$i, j = 0, \dots, n+1,$	(2.5)
	$d_i \le y_i \le Q,$	$i=0,\ldots,n+1,$	(2.6)
	$x_{ij} \in \{0, 1\},\$	$i, j = 0, \dots, n+1.$	(2.7)

Pedro Munaria et. al., A generalized formulation for vehicle routing problems, arXiv:1606.01935

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